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DOES THE STUDY OF MATHEMATICS TRAIN THE MIND SPECIFICALLY OR UNIVERSALLY?

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Education is or at least aims to be a conscious process and a purposive undertaking. To teach anything we must first know what purpose is to be served by it and how it must be taught so that that purpose will be served. As there are many subjects which might be studied and many ways in which each one of them might be presented our first and continuing duty is to select from the whole number of possible subjects those few which are indispensable for the purposes of life and when we have done that we must next select from the many possible ways of studying these subjects those few ways of approaching them which are likely to lead to valuable results.

Now, why should one study anything? As nearly as I can discover there are three answers which are given to this question. First, we must study subects because we owe it to them to do so. It is a debt of honor, of reverence, of obeisance, or worship which we should pay them. We do not study them for what they do for us or what they will enable us to do. They are the ends. We are the means. This is subject worship, a kind of liturgical devotion which we are told we must pay to science, literature, mathematics, philosophy when they are hypostatized into self-existing realities. Its favorite call to prayer is science for the sake of science, literature for the sake of literature, knowledge for the sake of knowledge, and art for

art's sake. This is a peculiarly inhuman belief which annually requires the sacrifice of hecatombs of young lives. It seems to us to be just as idolatrous to worship the creations of men's minds as to worship the creations of men's hands. We are recommended to beware of idols. The creator is more to be revered than his creation. When the creation is ascribed virtue in itself, the proper relations are reversed. Knowledge, art, science, literature, philosophy and mathematics exist for man's sake and not he for them. The question always is, what are they to him, what can he make out of them, what can he do with them? Knowledge cannot be its own end. It must be for something. It must perform some work, must offer some assistance, must serve some human purpose. We may take it on credit but the time must come when it wall pay some sort of dividends. If it does not, it is simply useless and unmeaning. It makes no difference in a world in which only such things are regarded as real as make a difference.

The second reason for studying anything is that we cannot get along without it. It is an indispensable aid to us in doing our work. It may serve us in many ways, but we want it because in days to come we shall use it. It is because we are going to read that we study reading, are going to write that we study writing, are going to use geography and history, literature and science as long as we live that we study geography, history, literature and science, and the parts of these studies which are outworn or have no definite utility we omit, giving our attention exclusively to those aspects of them which have abiding value. According to this view studies are for use and education is preparatory. There are so many difficult things that each one of us must know how to do in order to get on with nature and with our fellowmen, that the whole of life is not sufficient for us to learn them. All that we can do in youth is to master the beginnings of a few of the great human operations. Advanced life must help us to perfect our knowledge of them. From this point of view it is immeasurably important that we do not waste our time upon studies or parts of studies which we cannot use in after years and immeasurably important that we study the subjects that have definite utility in such ways that we will go on using them and increasing our mastery of

them through the years that are to come. The school, then, exists to provide special opportunities for us to become acquainted with the first stages of our life business and must introduce us to it in such a way that we shall, from the first, appreciate its meaning and perform it with a growing interest and an expanding sense of its worth, so that when our school days are over we shall know that our education has but begun and will go on applying and using and perfecting our skill in the great arts of which it has taught us the fundamentals as long as we may live. Education, according to this view, is specific throughout. Its purpose is to enable the student to acquire the beginnings of certain indispensable forms of human skill without which he cannot be a society-supporting unit in a world in which men must live and let live and help themselves and each other in doing so. Every form of skill that we attempt to teach him gets its place in the school program solely because he cannot live a civilized life without practicing it. Traditional reasons are not a sufficient warrant for teaching anything. The course of study is to be made with reference to the future, not because of veneration for the past nor because of blind adherence to the prevailing practice of to-day. The training of the young is so serious a responsibility that it must be made throughout a conscious undertaking. Their time must not be wasted and their future must not be trifled away. Nothing must be attempted in their education without demonstrable reasons for attempting it. Few men who have not followed closely the advances which have been made in the science of education in recent years know how completely present-day educational theory differs from the crude traditionalism of an earlier time. The new efficiency program which schools are trying to put into practice now is first to analyze the habits we want the young to form, to set up specific aims by whittling our purposes to the finest point in helping them to form them, and to measure carefully the results which are brought about by instruction. The effort of to-day is to do away with aimless routinary education, by substituting for it an intelligent procedure which shall be as rational as our present knowledge demands and warrants.

The third reason which has been assigned for studying anything is not that we owe it to the thing we are invited to study

to show it this tribute of respect and adoration, or that we shall need it in order to do our part in carrying on the unfinished business of the race. The third reason for studying certain subjects is that they perfect the mind and make it a better mind than it was before. The main province of the school according to this view is to train the mind not by putting it to work upon the matters it will have to work upon as long as it is a living mind, but to prepare it to work upon these matters by working upon others. This might be called indirect education because it maintains that the best way to learn to do one thing is to learn to do another. But if the theory were put as baldly as that, no one would believe it. It is couched in a more seductive form. Certain studies we are told teach us not only to work with their content, but to work with every content. They have far reaching effects, they enable us to do everything we undertake better because we have pursued them. Much of our learning we must get at retail, acquiring it painfully process by process and never getting any more than we bargain for, and mostly less. I have never heard teachers of history, for example, say that studying history teaches anything but history, or teachers of Spanish that studying Spanish teaches anything but Spanish. Just recently we have heard from eminent physical trainers that military training teaches military training and contributes nothing that makes for bodily well-being but much that harms it. But I have heard teachers of Greek and Latin and French and German say that the study of their subject is not intended to teach Greek or Latin or French or German. The study of their subjects is intended to improve the faculties of the mind. They claim to educate by wholesale, to give instruction in preferred subjects. They do not set out to teach their students the subjects which they study; they teach them, they say, something far more valuable. There are many variants of this claim and as nearly as I can discover no one knows exactly what they mean. I heard one man say in a discussion a while ago that he took it as established that we must sharpen an axe on some other material than that which we proposed to cut with it, likening the mind to an axe and the studies which he espoused to a grindstone, but the mind which God gave us is a pretty sharp instrument from the beginning, and we do not need to get inside it

to do any burnishing or repair work there. I find in Professor Keyser's interesting discussion of mathematics some statements which are puzzling and very hard to make out. "The science," he says, "is no catholicon for mental disease. There is no power for transforming mediocrity into genius. It cannot enrich where nature has impoverished. It makes no pretense of creating faculty where none exists, of opening springs in desert minds. . . . The great mathematician, like the great poet or the great administrator, is born. My contention shall be that where the mathematic endowment is found there will usually be found associated with it, as essential implications of it, other endowments in generous measure, and that the appeal of the science is to the whole mind, direct no doubt to the central powers of thought, but indirectly through sympathy of all, rousing, enlarging, developing, emancipating all, so that the faculties of will, of intellect and feeling learn to respond, each in its appropriate order and degree, like the parts of an orchestra to the 'urge and ardor' of its leader and lord." If the study of mathematics can do that or anything like that it is clear that we must all study mathematics, for though many of us have little occasion to use more than the merest elements of this great science, we all want our minds aroused, enlarged, developed and emancipated so that the faculties of will and intellect and feeling will respond. But is Professor Keyser not claiming too much? If mathematics could indeed do these things would it not be the philosophers' stone? And if it can do these things I trust it will not be thought impertinent to ask why it has not done them. Surely no greater harm can be done to any science than to overestimate its claims and mistake its nature and no greater harm can be done to the young than to submit them to a laborious and time-consuming discipline if we are not certain that that discipline can accomplish what we claim that it can accomplish.

Let us stop long enough to understand each other. The question which we are to consider is not the question of the value of mathematics; nobody doubts its value to anyone who has occasion to use it. The quetsion we are to consider is whether it is to be regarded as unlike other studies which are valuable to those who use them and not of much account to those who do not, but is instead a preferred study which is to be pursued not

for the sake of what we can do with it, but for the sake of what it will do to us. The value of mathematics as a tool, a human device for doing its part of the work of the world, is not disputed—it never has been. The value of mathematics as a universal discipline is not proven; it is disputed. Does learning mathematics teach mathematics as Robert Browning said that "learning Greek teaches Greek and nothing else; certainly not common sense if that have failed to precede the teaching"? Or does learning mathematics teach reasoning in general, not to say anything of its power to arouse, enlarge, develop, and emancipate the faculties of will and feeling?

If we go back to the Greeks who invented this great science we find them taking pains to put limits to their reliance upon it: in the "Memorabilia" of Xenophon we are told that Socrates had very decided views as to the value of geometry. "Everyone (he would say) ought to be taught geometry so far, at any rate, as to be able if necessary, to take over or part with a piece of land, or to divide it up or assign a portion for cultivation, and in every case by geometric rule. That amount of geometry was so simple indeed and easy to learn, that it only needed ordinary application of the mind to the method of mensuration, and the student could at once ascertain the size of the piece of land, and with the satisfaction of knowing its measurement depart in peace. But he was unable to approve of the pursuit of geometry up to the point at which it became a study of unintelligible diagrams. What the use of these might be he failed, he said, to see; and yet he was not unversed in these recondite matters himself. These things, he would say, were enough to wear out a man's life and to hinder him from many more useful studies. . . . Socrates inculcated the study of reasoning processes, but in these, equally with the rest, he bade the student beware of vain and idle over-occupation. Up to the limit set by utility he was ready to join in any investigation and to follow out an argument with those who were with him; but there he stopped." [Xenophon: "Memorabilia," IV., 7.]

This passage is thoroughly in keeping with Cleanthes's statement that Socrates cursed as impious "him who first sundered the just from the useful." Socrates's disciple Plato made a larger use of mathematics in the course of study which he out-

lined for the few selected youths whom he proposed to train to be philosopher-kings in the Republic of his vision. You will remember that he prescribed for them a ten years course in arithmetic, geometry, astronomy, and music because these studies lead naturally to reflection, but seem never to have been rightly used. The example which he gives of the way in which he would use these studies shows that he did not rely upon such a knowledge of them as our students are invited to get to lead his disciples to reflection. "When there is some contradiction always present and one is the reverse of one and involves the conception of plurality, then thought begins to be aroused within us and the soul perplexed and wanting to arrive at a decision, asks: 'What is absolute unity?' This is the way in which the study of the one has a power of drawing and converting the mind to the contemplation of true being. You are right, he said; the observation of the unit does certainly possess this property in no common degree, for the same thing presents at the same moment the appearance of one thing and an infinity of things." ("Republic," 524 and 525.) Plato's study of arithmetic is undertaken to consider the nature of numbers, and his geometry, the nature of space. It is intended to lead the student to discover the reality of mind, to know himself the thinker, not the science of mathematics. Will ten years of such study give him a trained mind? These studies, he says, are "useful, that is, if sought after with a view to the beautiful and good; but if pursued in any other spirit, useless. . . . Do you not know that this is only the prelude of the actual strain which we have to learn? For you surely would not regard the skilled mathematician as a dialectician? Assuredly not, he said. I have hardly ever known a mathematician who was capable of reasoning."

We find Aristotle too declaring that "the man of education will seek exactness so far in each subject as the nature of the thing admits, it being plainly much the same absurdity to put up with a mathematician who tries to persuade instead of proving, and to demand strict demonstrative reasoning of a rhetorician. Now each man judges well what he knows and of these things he is a good judge: on each particular matter he is a good judge who has been instructed in it, and in a general way the

man of general cultivation." ("Ethics," 1094*b*.) But this general cultivation is to be gotten by familiarity with many subjects not from the study of any one subject. "The capacity of receiving knowledge is modified by the habits of the recipient mind. For as we have been habituated to learn, do we deem that everything ought to be taught, and the same object presented in an unfamiliar manner, strikes us not only as unlike itself, but from want of custom as comparatively strange and unknown. . . . We ought therefore to be educated to the different modes and amount of evidence which the different objects of our knowledge admit." ["Metaphysics," II., 3.] There is no recognition of mathematics as teaching more than mathematics here. These Greeks do not rely upon it as a training in universal reasoning.

No such claim is made for the study until the faculty psychology brought faculty education in its train some time about the beginning of the eighteenth century. Faculty psychology is everywhere recognized as false doctrine since the criticism of Herbart gave it its deathblow in the early years of the nineteenth century. But faculty education still remains, though the psychologists tell us that there are no faculties to be educated. This of itself is a curious commentary upon the unscientific character of our education; but before I consider the claim that mathematics trains the faculty of reasoning I want to point out that there have from its first appearance as a philosophy of education been almost or quite as many competent critics of this doctrine as upholders of it.

I trust I shall not unduly tax your patience, if I refer to that remarkable article "On the Study of Mathematics as an Exercise of Mind" which Sir William Hamilton published in 1836. Professor Keyser calls it "Sir William Hamilton's famous and terrific diatribe against the science," but opinions of mathematicians seem to differ about it, for Professor Young finds it instructive to the teacher of mathematics and regards it as "a pity that more such criticisms are not made." Whatever else Sir William Hamilton's essay may be, it is not a diatribe against the science of mathematics. He says expressly: "In the *first* place that the question does not regard the *value of mathematical* SCIENCE *considered in itself, or in its objective results,*

but the *utility of mathematical* STUDY, that is, *in its subjective effect, as an exercise of mind;* and in the *second,* that the expediency is not disputed, of leaving mathematics as a co-ordinate, to find their level among the other branches of academical instruction. It is only contended that they ought not to be made the principal, far less the exclusive object of academical encouragement. We speak not now of professional but of liberal education; not of that which considers the mind as an instrument for the improvement of science, but of this which considers science as an instrument for the improvement of mind. Of all our intellectual pursuits the study of the mathematical sciences is the one whose utility as an intellectual exercise when carried beyond a moderate extent, has been most peremptorily denied by the greatest number of the most competent judges; and the arguments, on which this opinion is established have hitherto been evaded rather than opposed." If anyone has any doubt about the number of opinions which he musters to support his contention "that the tendency of a too exclusive study of these sciences is absolutely to disqualify the mind for observation and common reasoning" he has only to consult the article to learn how numerous they are. And I do not think it is fair to refute this article by ascribing it to "jealousy, vanity, and parade of learning," or to set it aside by declaring "that Hamilton by studied selections and omissions deliberately and maliciously misrepresented the great authors from whom he quoted . . . d'Alembert, Blaise Pascal, Descartes and others, distorting their express and unmistakable meaning, even to the extent of complete inversion."* It is easy to make charges against men who quote. That is a familiar line of attack. They can be charged with quoting what they should not have quoted, or of not quoting what they should have quoted. Such charges divert attention from what one has quoted but they do not answer it. The question is not whether Sir William Hamilton quotes less than there is to quote—everyone who quotes at all selects what he will quote—and the question is not whether the statement which he quotes in any given case is the average statement of its author upon the subject or the final result of a lifelong consideration of it. These men may have said other things at other

* Keyser, "Mathematics," pp. 23, 24, Columbia University Press. 1907.

times and in other places. They could hardly have been mathematicians without doing so. The question is whether they also at any time or in any place said what Sir William Hamilton quotes them as saying. Did d'Alembert ever say "we shall content ourselves with the remark, that if mathematics (as is asserted with sufficient reason) only make straight the minds which are without a bias, so they only dry up and chill the minds already prepared for this operation by nature." ("Melanges," IV., p. 184, 1763.) It is plain that if he contented himself with that remark we must be contented with that remark as coming from him. And did Descartes say that "the study of mathematics principally exercises the imagination in the consideration of figures and motions" ("Lettres," p. I.–XXX.) and to another correspondent "that part of the mind, to wit, the imagination, which is principally conducive to a skill in mathematics, is of greater detriment than service for metaphysical speculations" ("Epis.," p. II.–XXXIII.) and did Descartes's biographer, Baillet, write "It was now a long time since he had been convinced of the small utility of the mathematics especially when studied on their own account, and not applied to other things. There was nothing in truth which appeared to him more futile than to occupy ourselves with simple numbers and imaginary figures, as if it were proper to confine ourselves to these trifles without carrying our view beyond. There even seemed to him in this something worse than useless. His maxim was that such application insensibly disaccustomed us to the use of our reason and made us run the danger of losing the path which it traces." And does his Life contain the statement that in a letter to Mersenne, written in 1630, M. Descartes recalled to him that he had renounced the study of mathematics for many years; and that he was anxious not to lose any more of his time in the barren operations of geometry and arithmetic, studies which never lead to anything important." And does the author of Descartes's Life in a later passage say "in regard to the rest of mathematics" (he has just been speaking of astronomy) "those who know the rank which he held above all mathematicians, ancient and modern, will agree that he was the man in the world best qualified to judge them. We have observed that after having studied these sciences to the bottom, he had re-

nounced them as of no use for the conduct of life and solace of mankind." ("La Vie de Descartes," I., pp. 111, 112, 225.) It is no answer to such citations to make a great bluster about other statements which might have been quoted and to draw back from these as though it were a profanation even to think of them. The question which must be faced is: Did d'Alembert and Descartes and Descartes's biographer ever at any time say these things? The one legitimate way to attack Sir William Hamilton's use of them as evidence is to deny that they are to be found in the writings of these men. That denial is not made and cannot be made. These are statements which d'Alembert, Descartes and Descartes's biographer made, and made in words which mean exactly what we have indicated and must be reckoned with.

The passage which is quoted from Pascal is quoted at length. In it Pascal says: "There is a great difference between the spirit of mathematics and the spirit of observation. In the former the principles are palpable but remote from common use; so that from want of custom it is not easy to turn our head in that direction; but if it be turned ever so little the principles are seen fully confessed, and it would argue a mind incorrigibly false to reason inconsequently on principles so obtrusive, that it is hardly possible to overlook them. But in the field of observation, the principles are in common use and before the eyes of all. We need not to turn our heads to make any effort whatsoever. Nothing is wanted beyond a good sight; but good it must be, for the principles are so minute and numerous that it is hardly possible but some of them should escape. The omission, however, of a single principle leads to error; it is, therefore, requisite to have a sight of the clearest to discern all the principles; and then a correct intellect to avoid false reasonings on known principles. All mathematicians would thus be observant had they good sight, for they do not reason falsely on the principles they know; and minds of observation would be mathematical could they turn their view toward the unfamiliar principles of mathematics. The cause why certain observant minds are not mathematical is because they are wholly unable to turn themselves toward the principles of mathematics; but the reason why there are mathematicians void of observation is that they

do not see what lies before them, and that accustomed to the clear and palpable principles of mathematics and only to reason after these principles have been well seen and handled they lose themselves in matters of observation where the principles do not allow of being thus treated. These objects are seen with difficulty; nay, are felt rather than seen, and it is with infinite pains that others are made to feel them if they have not already felt them without aid. They are so delicate and numerous that to be felt they require a very fine and a very clear sense. They can also seldom be demonstrated in succession as is done in mathematics, for we are not in possession of their principles, while the very attempt would of itself be endless. The object must be discovered at once by a single glance and not be a course of reasoning, at least up to a certain point. Thus it is rare that mathematicians are observant and that observant minds are mathematical because mathematicians treat matters of observation by rule of mathematics, and make themselves ridiculous by attempting to commence by definitions and by principles a mode of procedure incompatible with this kind of reasoning." ("Pensées de Pascal," p. 1, Article X.)

But Sir William Hamilton is not satisfied with this showing that in learning mathematics we do not learn to reason about all things but only about mathematics; he quotes from scores of other persons to the same effect. His argument is not met by Professor Young's statement, that as mathematics was then taught the subject had, as Sir William Hamilton contended, but small value, "but mathematics is no longer taught as a purely passive subject to-day." That may be true and it is good news if it is true, but Sir William's point is that mathematics cannot be taught in such a way as to enable the student who has studied it, no matter how diligently to reason well about everything. Its lessons have no such universal reference and its methods of reasoning no such universal applicability. The reasoning which life exacts of us is upon contingent matter, the reasoning to which mathematics habituates us is upon necessary matter. In mathematics the premises are given; in life for the most part they must be found. The question we try to answer in mathematics is, what conclusions follow from these premises; the question we are forced to answer in life is, of what principle

is this case an instance or under what principle does this particular belong?

The case against mathematics, not as a science but as a universal trainer of the mind, has become very much stronger since 1830 than it was in Sir William Hamilton's brilliant summary of it. To the crowd of witnesses whom he summoned the names of Huxley and Comte and many another leader of human thought must now be added. The breakdown and abandonment of the faculty psychology left the doctrine of faculty education literally without a leg to stand on. If instead of one memory we have as many memories as the things we remember we cannot train or develop *the* memory for there is none to train. If our nature is so economical that we forget all the things which we have no occasion to remember and remember only those things in which we have taken a lively interest or about which we have built up a net of associations then the way to develop one's memory is to make no effort to develop it, but to spend one's strength instead in finding reasons for being interested in the thing which we want to remember. Let the memory alone, take no memory training lessons, give up forever the notion that a memory ever existed outside of the world of fancy which could remember all things equally well, let the memory alone and give your whole attention to comprehending what you want to remember. That is all that you or anyone else can do. This, you see, requires us to shift our attention wholly from the mind to the content.

The same criticism applies to the training of the reason. No such faculty exists. We reason well about one interest and badly about another. Such a thing as an all round reasoner is not to be found. The agriculturalist reasons well about growing crops, the commission merchant knows more about how to sell them. The geologist reasons well about rocks, the biologist about vital processes, the lawyer about laws, the engineer about the strength of materials, the physician about diseases and the tax expert about the incidents of taxation. The United States wants 150,000 ship carpenters, house carpenters will not do. We are specialists all. The study of mathematics makes a specialist out of the man who pursues it as his life work. How can the same study that makes specialists out of adults make

generalists out of the young? When we study mathematics we learn to make analyses, but to analyze the mathematical "given" is not the same thing nor even the same sort of thing as to resolve an economic situation into its constituent elements or a historical period into the forces which are operating in it or a crime into the factors which indicate its authorship. There are many forms of analysis and only the man who is familiar with a given subject matter can resolve it into its parts. The same thing is true of inferences and of the tracing of relations. The type of analysis or inference which is valid in one field is not valid in another. The universe of facts is no snug-fitting box with interchangeable parts which we can put together and take apart in a few well-defined ways. It is infinitely complex and he who is being trained to operate any part of it must be familiar with the characteristics of his particular field of fact and the processes of manipulation which belong to it. "Going to the root of the matter," says Professor Dewey, in speaking of the doctrine of formal discipline, "the fundamental fallacy of the theory is its dualism; that is to say, its separation of activities and capacities from subject matter. There is no such thing as an ability to see or hear or remember in general; there is only the ability to see or hear or remember something. To talk about training a power mental or physical in general, apart from the subject matter involved in its exercise, is nonsense."

If we turn to the experimental studies which have been made upon this subject we must note that they were not undertaken to inquire whether the memory, or the imaginaion or the observation or the reason can be trained as a faculty. No one who is at all conversant with modern psychology takes that question with any seriousness whatever. Any investigation of it would be a mere waste of time.

Since the psychologists agree that we have a different memory for everything we remember, a different attention for everything to which we attend, a different imagination for everything we imagine, and a different reasoning for everything we reason about, why should there be any investigation to find out to what extent learning to do one thing will help us to do another? The answer is that though our acts are different some of them have common elements and call forth identical responses. If

we learn to count marbles we can count eggs, for the act is the same in both cases, but it does not follow that if we learn to count objects we can count abstractions; that is a new art and must be learned, nor does it follow that if we can count abstractions that we can successfully number objects. There is a great gulf fixed between theoretical and practical arithmetic and between theoretical and practical mathematics throughout. A banker friend of mine declares that counting money in a large bank is so different from counting money in a small bank that city banks hesitate to employ as assistants man who have been trained in country banks. There is much that is common to the two processes but there is at the same time so much that is different that training in one does not prepare for the other.

One who learns to drive a Packard car can drive a Stanley Steamer, that is, he can steer it, for he is only doing over again what he has already learned to do, but one who can adjust a Packard engine cannot adjust the engine in a Stanley Steamer without a special knowledge of that engine.

The ability to use the knowledge which we have acquired in one connection in another is sometimes said to be due to a transfer of training. Professor Dewey tells us that "in the literal sense any transfer is miraculous and impossible." What then does the transfer which is said to take place really mean? Learning to drive a Packard car enables one to drive a Stanley Steamer, because when we drive the Steamer we are simply doing over again what we have already learned to do. Nothing is transferred; instead an act we have already learned to perform is repeated, in a context very like the context in which it was learned. If we could transfer our training from one context to another quite freely we would not go on merely repeating what we have already learned. We would all become inventors. The fact that inventions are and always have been so rare shows quite clearly that we do not do that. We do over and over again what we have already learned to do; but within what limits do we repeat our familiar reactions? That is the question which the experimentalists are answering and their answers all show that the limits go but a little way beyond the lesson itself and that the range of its application is very narrow indeed.

Some of these experiments seek to determine the effects of training in mathematics upon the performance of other kinds of work. One of them is the series of tests conducted by Lewis at Dartmouth. Two test papers were prepared, one containing three originals in geometry, the other three questions in practical reasoning concerning the value of high-school education to the student and the community. Both papers were submitted to 24 different groups of high-school students. The results I give in Mr. Lewis's own words: "If we take the first five mathematical reasoners from each of the 24 groups, we have in all one hundred and twenty pupils most excellent in mathematical reasoning. Of this number 76 or 63 per cent. are at the foot of the practical reasoning series, conspicuous for their inefficiency in practical reasoning. Of the number of pupils at the foot of the mathematical reasoning series, 57, or 47 per cent. are conspicuous for their positions at the head of the practical reasoning series." To supplement this test the records of Dartmouth students in the classes in mathematics and in courses in law were compared. The results were much the same. "Fifty per cent. of the best students in law were conspicuous for their poor showing in mathematics and 42 per cent. of those poorest in law stood at the head of the series in mathematics."

More recently at the University of Illinois Dr. Rugg conducted a classroom experiment in which two groups, one of 413, and the other of 87 college students, were first measured for efficiency in the mental manipulation of spatial elements. The first group of 413 students then took a regular course in descriptive geometry during a college semester of 15 weeks. The other group of 87 college students had no such training during this interval. At the end of the 15 weeks both groups were again measured as they had been at the beginning to discover the effect of the course in descriptive geometry which the one group had taken and the other had not, upon specific abilities in the mental manipulation of spatial elements, (*a*) of a strictly geometrical type; (*b*) of a quasi-geometrical type; and (*c*) of a non-geometrical type. What was the result? Members of both the trained and the untrained group showed improvement in taking the test series a second time. But there were 44 per cent. more gainers in speed in the trained group than in the untrained and

nearly two thirds again as large a proportion of the trained group as of the untrained group gained in accuracy. Of the group that had the training not all gained and of the group that did not have the training a very large number gained as much as those who had had it.

How many individuals gained? "In 'attempts' 67.8 per cent. of the training group and 42.5 per cent. of the control group gain in 60 per cent. or more of the tests taken." That is 42½ folks out of every hundred who did not have the training took the tests as successfully as 68 out of every hundred who did have it. That is, the course seems to have been of some positive assistance in preparing only 25½ folks out of each hundred to take the test. To 32 out of every hundred who took it it was no help and 42½ of every 100 who took it got on just as well without it as with it, that is so far as attempts went. "In 'Rights' 72.7 per cent. of the training group and 31 per cent. of the control group gain in 60 per cent. or more of the tests taken." If 72.7 per cent. who took the training gained we may conclude that 27 out of every hundred who took it did not gain and as 31 per cent. of those who did not have it did as well as those who did have it only 42 out of every hundred became more accurate because of it, while 58 did not, thus you see the chances seem to be about 6 to 4 against expecting anything in the way of general training, that is training which is not strictly specific from such a course. On Dr. Rugg's showing the dice are loaded against every student who takes this course for general training.

It is true, as he points out, that more of those who took the training gained than of those who did not, but a considerable number of those who took it did not gain, and a very considerable number of those who did not take it gained. So to gain it is not necessary to take it and if one does take it there is no certainty that he will gain.

These are his figures but this is not Dr. Rugg's conclusion. His conclusion is that these results supply confirmatory evidence of the "transfer of training," though, as he says, his data do not of course establish conclusively the possibility of transfer. It is not the possibility of transfer but rather the actuality of transfer that concerns educators. His results, like

those of all the experimental studies I have seen, seem to me to assist materially in establishing the fact that we cannot any longer make a philosophy of education out of the doctrine of formal discipline, and they very positively confirm the suspicion with which any such attempt must be met. The burden of proof rests upon those who uphold this theory. It has never been proven, and until it is proven it is mere conjecture wholly insufficient as a theory of instruction.

Education is too serious a business to be allowed to proceed upon chances which are mathematically known to be against the student. Some of those who have investigated the question whether training is transferred declare that it is not. Some affirm that under certain conditions it is sometimes and in some degree; but even when they declare that it is transferred the evidence of transfer is so inconclusive and the amount of the so-called transfer is so slight and the expectation of it so uncertain, that it is the part of wisdom no longer to build houses of learning upon the shifting sands of this doctrine. The investigations have put a cloud upon the title of this theory of education which cannot be removed. It simply does not work. On the solid rock of specific education we can build and must build, for of the results of specific education we can be sure, but as for formal or general discipline, in the words of Professor Spearman "the great assumption upon which education has rested for so many centuries is now at last rendered amenable to experimental corroboration—and it proves to be false."

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